Objective: In this lesson, you will construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.

inscribed circle of the triangle - ______________________________________________________________

circumscribed circle of the triangle - __________________________________________________________

The Inscribed Circle of a Triangle

Watch the video, which demonstrates a traditional method for constructing the inscribed circle of a triangle.

Then scroll down and write the 5 steps on how to inscribe a circle in a triangle.

1) ______________________________________________________________________________________

2) ______________________________________________________________________________________

3) ______________________________________________________________________________________

4) ______________________________________________________________________________________

5) ______________________________________________________________________________________
Example

What are the steps for constructing an inscribed circle in \( \triangle ABC \) using only a compass and a straightedge?

**Step 1:** Place the compass needle on vertex \( A \), adjust the width of the compass to a medium setting, and draw one arc on line segment \( AB \) and one on line segment \( AC \).

**Step 2:** Place the compass needle on the point where the arc intersects line segment \( AB \) and draw an arc inside the triangle. Without changing the compass setting, place the compass needle on the point where the arc intersects line segment \( AC \); draw another arc intersecting the arc already inside the triangle. Label the point of intersection \( P \).

**Step 3:** Draw line segment \( AP \). This is the angle bisector of angle \( A \).

**Step 4:** Place the compass needle on vertex \( C \), adjust its width to a medium setting, and draw arcs intersecting line segment \( BC \) and line segment \( AC \).

**Step 5:** Place the compass needle on the point where the arc intersects line segment \( BC \), and draw an arc inside the triangle. Without changing its setting, place the compass on the point where the arc intersects line segment \( AC \); draw another arc intersecting the arc already inside the triangle. Label the point of intersection \( Q \).

**Step 6:** Draw line segment \( CQ \). This is the angle bisector of angle \( C \).

**Step 7:** Label the intersection of the angle bisectors \( M \). This is the center of the inscribed circle.

**Step 8:** Place the compass needle on \( M \), and draw two arcs on line segment \( BC \).

**Step 9:** Place the compass needle on one of the points where an arc intersects line segment \( BC \), and draw an arc outside the triangle; repeat from the point where the other arc intersects line segment \( BC \) to create two intersecting arcs outside the triangle. Draw a line from \( M \) passing through the point of intersection outside the triangle. This is a perpendicular line from \( M \) to line segment \( BC \). Mark the point where the line intersects line segment \( BC \), and label it \( N \).

**Step 10:** Place the compass needle on \( M \), set its width to \( N \), and draw a circle. This is the inscribed circle of triangle \( ABC \).
Circle $F$ is inscribed in $\triangle ABC$.

**Example**

Brian is waiting at a park across the street from the hockey arena for his dad to pick him up. It rained most of the morning, and the streets still have puddles. At which point should Brian stand to be as far as possible from each street to avoid getting splashed? Explain your answer.
The Circumscribed Circle of a Triangle

The distance between the center of the circle and each of the vertices will be ____________________________

Watch the video to learn a traditional method using a compass and a straightedge.

Then scroll down and write the 4 steps on how to circumscribe a circle on a triangle using just a compass and a straightedge.

1) ________________________________________________________________________________________

2) ________________________________________________________________________________________

3) ________________________________________________________________________________________

4) ________________________________________________________________________________________
Example

What are the steps for constructing a circumscribed circle for $\triangle ABC$ using only a compass and a straightedge?

Step 1: Place the compass needle on vertex $A$, and set the width to more than half the length of line segment $AB$. Draw one arc on each side of line segment $AB$ but not passing through line segment $AB$.

Step 2: Using the same compass width, place the compass needle on vertex $B$, and draw two arcs intersecting the arcs made in step 1.

Step 3: Draw a straight line connecting the pairs of intersecting arcs initiated from points $A$ and $B$. This line is the perpendicular bisector of line segment $AB$.

Step 4: Place the compass needle on vertex $B$ and set the width to more than half the length of line segment $BC$. Draw one arc on each side of line segment $BC$ but not passing through line segment $BC$.

Step 5: Using the same compass width, place the compass needle on vertex $C$, and draw two arcs intersecting the previous arcs in step 4.

Step 6: Draw a straight line connecting the pairs of intersecting arcs initiated from points $B$ and $C$. This line is the perpendicular bisector of line segment $BC$.

Step 7: Mark the point where the perpendicular bisectors of line segment $AB$ and line segment $BC$ intersect, and label it $D$.

Step 8: With the compass needle on $D$, set its width to any one of the vertices of the triangle and draw a circle. This circle is the circumscribed circle of the triangle.
Example

A spirit shop outside the hockey stadium sells merchandise representing the three school districts that built the arena. The entrances to the arena, the spirit shop, and the parking lot form the vertices of a triangle. The planning committee has decided to erect a memorial statue and would like it to be equidistant from the entrances to the arena, the spirit shop, and the parking lot. At what point should the statue be located? Explain your answer.

A Quadrilateral Inscribed in a Circle

\[ m \angle A + m \angle C = \underline{\quad} \]

\[ m \angle B + m \angle D = \underline{\quad} \]
Example

In quadrilateral \(QRST\), \(m\angle Q\) is 68°, \(m\angle R\) is \((3x + 40)°\), and \(m\angle T\) is \((5x - 52)°\).

What are the measures of \(\angle R\), \(\angle S\), and \(\angle T\)?

Write the numerical values in that order with the measures separated by commas.

\[\text{Step 1: } \angle R + \angle T = 180\]

\[3x + 40 + 5x - 52 = 180\]

\[x = \underline{\phantom{0000}}\]

\[\text{Step 2: Substitute the value of } x \text{ into } \angle R \text{ and } \angle T\]

\[\angle R = 3x + 40 = 3(24) + 40 = \underline{\phantom{000}}\]

\[\angle T = 5x - 52 = 5(24) - 52 = \underline{\phantom{000}}\]

\[\text{Step 3: } \angle Q + \angle S = 180\]

\[68 + \angle S = 180\]

\[\angle S = \underline{\phantom{000}}\]

Summary

If a circle is inscribed in a triangle, \[\underline{\phantom{0000000000000000000}}\]

If a circle is circumscribed about a triangle, \[\underline{\phantom{0000000000000000000}}\]

If a quadrilateral can be inscribed in a circle, \[\underline{\phantom{0000000000000000000}}\]